J. Eborn, K.J. Åström:
**Modeling of a Boiler Pipe with
Two-Phase Flow Instabilities.**
Modelica Workshop 2000 Proceedings, pp. 79-88.

Modeling of a Boiler Pipe with Two-Phase Flow Instabilities

Jonas Eborn and Karl Johan Åström

Department of Automatic Control
Lund Institute of Technology, Lund, Sweden
October 13, 2000

Abstract: Tubes with boiling are common elements of many processes. They appear in steam generators and refrigerators and many other systems. The behavior of such systems is complicated and many physical phenomena are involved. It has for example been observed that different types of instabilities can occur. In this paper we will discuss modeling of tubes with boiling. As an application we will discuss an instability phenomenon due to pressure oscillations that has been observed experimentally in many different situations. We will first derive a simple analytical model which is able to capture the oscillations qualitatively. The simple model also gives insight into the mechanisms that generate the oscillations. A more complicated model is then built using a recently developed model base library in Modelica. A comparison between the simple and the complicated model is also given.

1. Introduction

Evaporation of fluids flowing through a tube is common in many processes. It is a key element in steam generators, refrigerators and many other systems. The physical phenomena during evaporation is quite complicated. Both the dynamics and fluid property relations are highly non-linear and key quantities like the dry-out point or the amount of superheat are difficult to measure. Many factors contribute to making these processes hard to control. It has for example been observed that different types of flow instabilities/oscillations can occur, see (1, 4, 8).

In this paper we will discuss modeling of tubes with boiling. As an application we will discuss an instability phenomenon due to pressure oscillations that has been observed experimentally in different situations, e. g., in (5). Modeling of such systems is usually done by "brute-force", using CFD code with high discretization. In this paper we take a different approach.

Within the framework of a European collaboration a new modeling language, called Modelica, has been developed. Modelica is based upon the experiences of the members of the Modelica Design Group and is aiming at becoming a standard for equation-based continuous-time and hybrid modeling. As a part of the effort some Modelica base libraries for applications within different domains have been developed, among these a thermo-hydraulic base library, see (7). This library is used here to build a discretized model of a boiling tube.

First a simple, low-order analytical model is derived from first principles. The simple model is able to capture the oscillations that have been observed experimentally. It can also give insight into the mechanisms that generate the oscillations. Then a more complex model is built using the thermo-hydraulic base library in Modelica. Simulations of the complex model shows that it gives
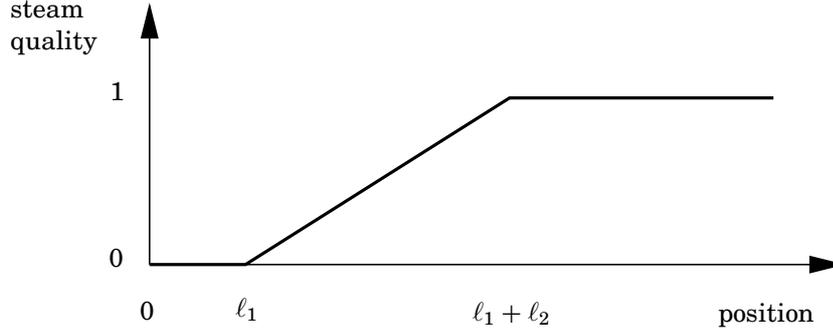
Fig. 1 Distribution of vapor along a heated tube.

results comparable to the simplified model and also close to experimental data. But the complex model also has a richer behavior. Comparisons between the two models are given.

2. Simple Physical Analysis

When liquid streams through a heated tube it is first heated to the boiling temperature, when boiling occurs there is a mixture of liquid and gas and finally there is only gas which is heated. This is illustrated schematically in Figure 1. The heating zone is from 0 to ℓ_1, the boiling zone from ℓ_1 up to $\ell_1 + \ell_2$ and after this point there is no liquid left in the tube. If the flow is sufficiently fast the liquid does not spend enough time in the tube for all liquid to evaporate and then there are only two zones. If the flow rate is even higher boiling may not even start.

Two phase flows are quite complicated. Here we will first start with a simplified, homogeneous model. Let P be the power per unit length supplied to the tube, let m be the mass flow rate, let h_{in} be the enthalpy of the liquid at the entrance of the tube, h_l the liquid enthalpy at boiling temperature and $h_c = h_v - h_l$ the difference between the enthalpy of the vapor and the liquid. Moreover let L be the length of the tube and ℓ_1 and ℓ_2 be the length of the heating and boiling zones. If we assume stationary conditions, a global energy balance gives

$$m(h_l - h_{in}) = P\ell_1$$
$$mh_c = P\ell_2 \tag{1}$$

The condition for having complete boiling is that $\ell_1 + \ell_2 < L$. Neglecting the enthalpy increase of the vapor we get

$$m < m_c = \frac{PL}{h_v - h_{in}} \tag{2}$$

where m_c is the critical mass flow rate.

The pressure drop is

$$\Delta p = \frac{k}{2}\overline{\rho v^2} = \frac{k}{2A^2}\overline{\frac{m^2}{\rho}} \tag{3}$$

To determine the pressure drop we thus have to calculate the average $\overline{1/\rho}$. To calculate the average we will consider three separate cases.

Complete Boiling: In this case all phases are present. In (2) it was shown that in steady state the mass ratio of the vapor in a heated tube is piecewise linear. If we assume that this profile is a good approximation also in the transient stage we can assume that the volumity, $v = 1/\rho$, is an affine function in the boiling zone, i.e.

$$v = \frac{\xi}{\ell_2}v_v + \frac{\ell_2 - \xi}{\ell_2}v_l$$

where $0 \leq \xi \leq \ell_2$ and the origin is at the start of boiling. Hence

$$\frac{\overline{1}}{\rho} = \overline{v} = \frac{1}{L}\left(\int_0^{\ell_1} v_l d\xi + \int_0^{\ell_2}\left(v_l + \xi\frac{v_v - v_l}{\ell_2}\right)d\xi + (L - \ell_1 - \ell_2)v_v\right)$$
$$= \frac{\ell_1}{L}\frac{1}{\rho_l} + \frac{\ell_2}{2L}\left(\frac{1}{\rho_v} + \frac{1}{\rho_l}\right) + \frac{L - \ell_1 - \ell_2}{L}\frac{1}{\rho_v} \tag{4}$$

It follows from Equation (1) and Equation (2) that

$$\frac{\ell_1}{L} = \frac{h_l - h_{in}}{h_v - h_{in}}\frac{m}{m_c} \qquad \frac{\ell_2}{L} = \frac{h_v - h_l}{h_v - h_{in}}\frac{m}{m_c}$$

Introducing $x = m/m_c$ into (4) we find that

$$\frac{\overline{\rho_l}}{\rho} = (1 - x)\frac{\rho_l}{\rho_v} + x\frac{h_l - h_{in}}{h_v - h_{in}} + \frac{x}{2}\frac{h_v - h_l}{h_v - h_{in}}\frac{\rho_l + \rho_v}{\rho_v}$$

Partial Boiling: In this case there is only a heating zone and a boiling zone. The flow at the exit of the tube consists of a mixture of both vapor and liquid. We have $\ell_1 < L = \ell_1 + \ell_2$ and we get

$$\frac{\overline{1}}{\rho} = \overline{v} = \frac{1}{L}\left(\int_0^{\ell_1} v_l d\xi + \int_0^{\ell_2}\left(v_l + a_r\xi\frac{v_v - v_l}{\ell_2}\right)d\xi\right)$$
$$= \frac{\ell_1}{L}\frac{1}{\rho_l} + \frac{\ell_2}{2L\rho_l}\left(2 + a_r\left(\frac{\rho_l}{\rho_v} - 1\right)\right) \tag{5}$$

where a_r is the mass fraction of vapor at the tube outlet. Neglecting the energy increase in the pure vapor phase a global energy balance gives

$$ma_r(h_v - h_l) = P\ell_2 = P(L - \ell_1) = m_c(h_v - h_{in}) - m(h_l - h_{in})$$

Combining this with Equation (2) we find

$$a_r = \frac{m_c(h_v - h_{in}) - m(h_l - h_{in})}{m(h_v - h_l)}$$
$$\frac{\ell_2}{L} = \frac{m}{m_c}\frac{h_v - h_l}{h_v - h_{in}}a_r = 1 - \frac{m}{m_c}\frac{h_l - h_{in}}{h_v - h_{in}}$$

Inserting this into (5) we get

$$\frac{\overline{\rho_l}}{\rho} = x\frac{h_l - h_{in}}{h_v - h_{in}} + \frac{1}{2}\left(1 - x\frac{h_l - h_{in}}{h_v - h_{in}}\right)\left(2 + \frac{h_v - h_{in} - x(h_l - h_{in})}{x(h_v - h_l)}\frac{\rho_l - \rho_v}{\rho_v}\right)$$

No Boiling: In the case where there is no boiling we have

$$\frac{\overline{\rho_l}}{\rho} = \frac{\rho_l}{\rho_l} = 1$$

Summarizing the different cases we find that the pressure drop is given by

$$\Delta p = \frac{km^2}{2A^2\overline{\rho}} = \frac{km_c^2}{2A^2\rho_l}f\left(\frac{m}{m_c}\right) \tag{6}$$

where the function $f = x^2\rho_l/\overline{\rho}$ is given by

$$f(x) = \begin{cases} x^2a_3 + x^3\left(a_1 + \dfrac{a_2}{2}(a_3 + 1)\right) - a_3 & \text{for } 0 \leq x < 1 \\[3mm] x^2\left(xa_1 + \dfrac{1 - a_1x}{2}\left(2 + \dfrac{1 - a_1x}{a_2x}(a_3 - 1)\right)\right) & \text{for } 1 \leq x < \dfrac{1}{a_1} \\[3mm] x^2 & \text{for } x \geq \dfrac{1}{a_1} \end{cases} \tag{7}$$
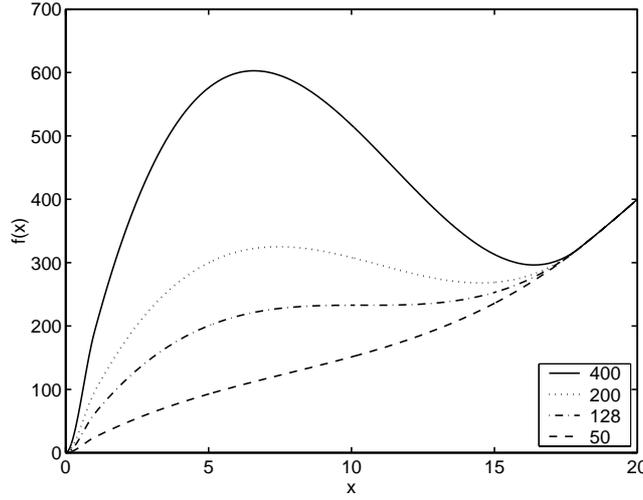
Fig. 2 The function f (normalized pressure drop) for different ρ_l/ρ_v.

where the coefficients a_i are given by

$$a_1 = \frac{h_l - h_{in}}{h_v - h_{in}} \qquad a_2 = \frac{h_v - h_l}{h_v - h_{in}} \qquad a_3 = \frac{\rho_l}{\rho_v} \tag{8}$$

Figure 2 shows that the curve has a negative slope for certain values of the ratio ρ_l/ρ_v. When this occurs the function f will also have non-trivial extrema.

2.1 *Conditions for local extrema*

As can be seen in Figure 2 the pressure drop function (7) can have negative slope for certain values of ρ_l/ρ_v. The local maximum and minimum will, for reasonable values of a_i, always occur in the partial boiling region, $1 \le x < 1/a_1$. By differentiating the expression for f in this region and setting it equal to zero we obtain

$$f'(x) = 3x^2\left(\frac{a_1^2(a_3-1)}{2a_2}\right) + 2x\left(1 - \frac{a_1(a_3-1)}{a_2}\right) + \frac{a_3-1}{2a_2} = 0$$

$$\Leftrightarrow 3(xa_1)^2 + 4(xa_1)\left(\frac{a_2}{a_1(a_3-1)} - 1\right) + 1 = 0$$

Solving this equation we find that the local extrema occur at

$$x_{1,2} = \frac{2}{3a_1}\left(1 - \frac{a_2}{a_1(a_3-1)}\right) \pm \frac{1}{a_1}\sqrt{\frac{4}{9}\left(1 - \frac{a_2}{a_1(a_3-1)}\right)^2 - \frac{1}{3}} \tag{9}$$

Examining the second derivative shows that it is negative for small x and becomes positive above $x \approx 2/(3a_1)$, indicating a maximum for the smaller x in (9) followed by a minimum. The condition for having two separate extrema is

$$1 \le \frac{4}{3}\left(1 - \frac{a_2}{a_1(a_3-1)}\right)^2$$

$$a_3 \ge a_3^c = 1 + \frac{a_2}{a_1}(4 + 2\sqrt{3}) \tag{10}$$

The solution to this inequality shows that over a certain density ratio, ρ_l/ρ_v, the conditions for local extrema are fulfilled and pressure-drop oscillations can occur. The critical density ratio (10) depends on a_1, a_2 and thus on the amount of sub-cooling. Analyzing the case when there is no sub-cooling ($a_1 = 0$) shows that the pressure drop is strictly increasing and thus there are no local extrema. For the conditions in Figure 2 the critical density ratio is $\rho_l/\rho_v = 127.89$.

Summarizing we find that if $a_3 = \rho_l/\rho_v$ fulfills condition (10) then the function f has extrema in (9) and the slope of f between the extrema is negative. The extrema will coincide if $\rho_l/\rho_v = a_3^c$. The difference $x_2 - x_1$ increases with increasing ratio ρ_l/ρ_v.

3. Analysis of Oscillations

Having obtained a model for the pressure drop we will now investigate some interesting dynamical phenomena. Figure 3 shows a schematic diagram of a boiling channel with a surge tank. This corresponds to the experimental configuration used in several experiments, see (5).

Let m_0 and m denote the mass flow rate in and out of the surge tank respectively. Let p be the pressure in the surge tank and p_0 the external pressure. The system can be described by mass and momentum balances. The variables p and m are chosen as states variables. The equations for these states are derived from a momentum balance for the heater tube and a mass balance for the surge tank:

$$\frac{dm}{dt} = \frac{A}{L}(p - p_0 - \Delta p) = \frac{A}{L}\left(p - p_0 - \frac{km_c^2}{2A^2\rho_l}f\left(\frac{m}{m_c}\right)\right)$$

$$\rho_l \frac{dV}{dt} = m_0 - m$$

where A is the cross-section of the tube and L is the length of the tube.

The pressure in the tank is given by the ideal gas law. By differentiating this equation we get an equation for dp/dt

$$p(V_t - V) = p_0 V_0 \quad \xrightarrow{\frac{d}{dt}} \quad \frac{dp}{dt}(V_t - V) = p\frac{dV}{dt}$$

Fig. 3 Schematic of a boiling channel.

Solving for dp/dt and eliminating $V_t - V$ we get

$$\frac{dp}{dt} = \frac{p}{V_t - V}\frac{dV}{dt} = \frac{p^2}{\rho_l p_0 V_0}(m_0 - m)$$

The system is thus described by the second order differential equation system

$$\frac{dm}{dt} = \frac{A}{L}\left(p - p_0 - \frac{km_c^2}{2A^2\rho_l}f\left(\frac{m}{m_c}\right)\right)$$

$$\frac{dp}{dt} = \frac{p^2}{\rho_l p_0 V_0}(m_0 - m) \tag{11}$$

The equilibrium of the equation system (11) is given by

$$m = m_0, \qquad p = p_0 + \frac{km_c^2}{2A^2\rho_l}f\left(\frac{m_0}{m_c}\right)$$

3.1 *Normalization*

Introduce the time constants

$$T_{ma} = \frac{\rho_l V_0}{m_0} \qquad \text{and} \qquad T_{mo} = \frac{m_0 L}{A p_0}$$

which are associated with the mass balance of the surge tank and the momentum balance of the heating tube respectively. Notice that $\rho_l V_0$ is the mass of the liquid in the volume V_0 and m_0 is the mass flow rate. Similarly, note that $m_0 L$ is the momentum of the fluid in the tube and $A p_0$ is the force acting on the fluid.

Fig. 4 Simulated limit cycle of pressure drop oscillations.

By then introducing the scaled time τ defined by

$$\tau = \sqrt{\frac{1}{T_{ma}T_{mo}}}t = \sqrt{\frac{Ap_0}{\rho_l V_0 L}}t$$

the equation system becomes

$$\frac{dx}{d\tau} = \alpha(y - 1 - \beta f(\gamma x))$$

$$\frac{dy}{d\tau} = \frac{1}{\alpha}(1 - x)y^2$$

(12)

where

$$\alpha = \sqrt{\frac{\rho_l V_0 A p_0}{L m_0^2}} = \sqrt{\frac{T_{ma}}{T_{mo}}}, \qquad \beta = \frac{k m_c^2}{2A^2 \rho_l p_0} = \frac{\Delta p_c}{p_0}, \qquad \gamma = \frac{m_0}{m_c}$$

and Δp_c is the stationary pressure drop when $m = m_0 = m_c$.

If the simplified model (12) is used the system is thus characterized by four parameters only; the ratios of densities, ρ_v/ρ_l, time constants, T_{ma}/T_{mo}, pressures, $\Delta p_c/p_0$, and mass flows, m_0/m_c.

3.2 *Linearization*

The equilibrium values of the normalized variables are

$$x = 1$$

$$y = y_0 = 1 + \beta f(\gamma)$$

If we choose $u = \Delta m_0/m_0$ as an input the linearized system of equations at the equilibrium is

$$\frac{d}{dt}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -\alpha\beta\gamma f'(\gamma) & \alpha \\ -y_0^2/\alpha & 0 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} 0 \\ y_0^2/\alpha \end{pmatrix}u$$

Since the coefficients α, β and γ are positive it follows that the equilibrium is stable if $f'(\gamma)$ is positive and that it is unstable if $f'(\gamma)$ is negative.

3.3 *Simulation*

The simple model (11) can easily be coded in Modelica to examine the behavior of the model. The pressure drop function, f, is given as a Modelica function with four arguments, as seen in the
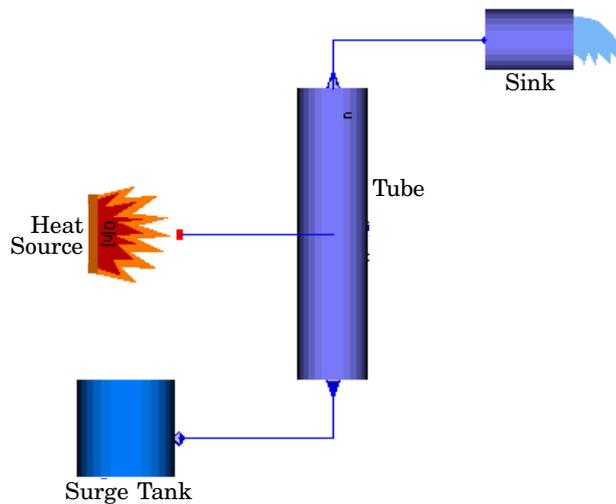
Fig. 5 Model diagram of the system.

code example below:

```
model evap "Evaporating tube with density-varying pressure drop"
  parameter SIunits.Power P(start=800);
    ⋮
  SIunits.Pressure dp;
  SIunits.Pressure p(start=3e5);
  SIunits.MassFlow m(start=0.024);
equation
  dp = k*mc*mc/(2*A*A*rl)*f(x=m/mc, a3=rl/rv, a1=a1, a2=a2);
  der(p) = p*p*(m0 - m)/p0/V0/rl;
  der(m) = A/L*(p - pe - dp);
end evap;
```

With parameters taken from (5) you obtain a similar limit cycle in pressure and mass flow as observed from their measurements. A phase plot of the limit cycle is shown in Figure 4. The pressure drop characteristic is drawn with a full line and the pressure in the surge tank is dashed. The period of the oscillation depends a lot on the volume of the surge tank, V_0, which is not stated in the reference.

4. More Complex Modelica Models

To verify the simplified model we can build a more complex, discretized model and compare the results. This is done using the Modelica base library for thermo-hydraulic models, ThermoFlow, see (7, 6). The basic components in this library are lumped and discretized control volumes containing the balance equations for mass, energy and momentum. Using these standard components, a system model as in Figure 5 is created with a surge tank and a discretized pipe. The pipe can be discretized to any degree, in the presented results $n = 10$ was used, which gave reasonable results. This complex model can exhibit a much more complicated behavior than the simplified one.

4.1 *One flow, multi-temperature model*
To obtain a model similar to the simplified model we use a lumped mass (pressure) balance and a discretized thermal (enthalpy) balance. In this way there is only one flow through the pipe instead

of n flows between the discretized volumes. Thus we can only obtain the type of pressure-drop oscillations seen with the simplified model. This mixed model is not available as a component in the library, but can be obtained by changing some of the basic components. Below are the central parts of the pipe model; BalanceTwoPort, ThermalModel and FlowModel. Only the essential code and changes to the classes are given.

For the mixed model to be correct, it is important to keep track of the mean thermal state, used in the ThermalModel below. It is also important to distribute the flow difference between inlet and outlet over the discretized energy balances, which can be seen in the BalanceTwoPort. Otherwise flow changes are concentrated in one section of the pipe, influencing the thermal state in that section too much.

```
partial model BalanceTwoPortSingleSpecial
  ...
  SIunits.MassFlowRate dm[1];
equation
  ...
  dm[1] = (mdot[1] - mdot[2])/n;
  edot[1] = a.q_conv;
  for i in 2:n loop    // Interpolated mass flow used for edot
    edot[i] = if mdot[1]-dm[1]*(i-1) > 0
      then (mdot[1]-dm[1]*(i-1))*h[i-1]
      else (mdot[1]-dm[1]*(i-1))*h[i];
  end for;
  edot[n+1] = if mdot[2]>0 then mdot[2]*h[n] else mdot[2]*b.h;
  for i in 1:n loop
    der(M[i]) = dm[1];
    der(U[i]) = edot[i] - edot[i+1] - p[1]*der(V[i]) + Q_s[i];
  end for;
end BalanceTwoPortSingleSpecial;


partial model ThermalModelSpecial
  replaceable model Medium = StateVariablesSpecial;
  extends Medium;
equation
  ...
  for i in 1:n loop    // thermal state equations
    km[i]*der(h[i]) = kh[1, i]*der(M[i]) + kh[2, i]*der(U[i]);
  end for;
  km[n + 1]*der(p[1]) = kp[1]*sum(der(M)) + kp[2]*sum(der(U));
  // Mean value of enthalpies in last component, gives mean thermal state
  h[n + 1] = h[1:n]*d[1:n]/sum(d[1:n]);
end ThermalModelSpecial;


model FlowModelTwoPortSingleSpecialDyn
  ...
equation
  G_norm[2] = if mdot[2] > 0
    then mdot[2]*mdot[2]/d[n+1]/A
    else -mdot[2]*mdot[2]/ddown/A;
  dG = G_norm[1]-G_norm[2] + dGdown;
  // This is the momentum balance equation
  L*der(mdot[2]) = dG + (p[1] - pdown)*A - sum(Ploss)/n*L*Dhyd*Pi;
end FlowModelTwoPortSingleSpecialDyn;
```

Simulating the mixed model gives similar results as the simplified model, see Figure 6. This verifies that the derivation of the pressure loss in the simplified model is correct. The simulation results are also qualitatively very close to the experimental results in (5).
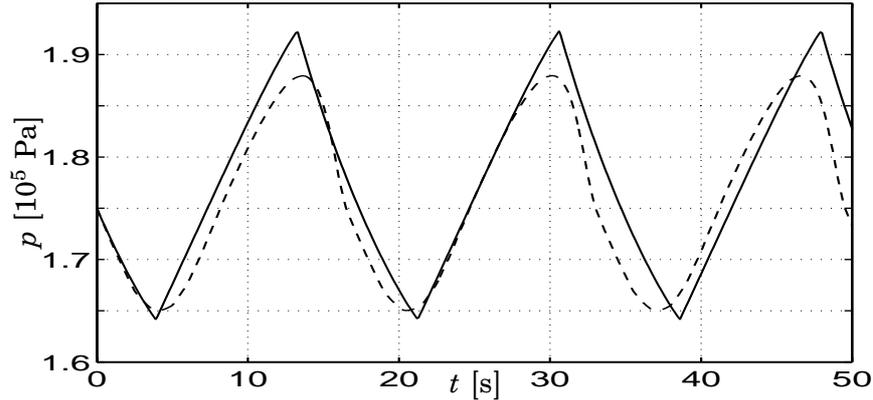
Fig. 6 Pressure drop oscillations with simplified (—) and complex model (– –).

4.2 *Fully discretized model*

The standard model of a discretized pipe with n mass balances (and flows) can also be used to study the problem with pressure oscillations. The results are however slightly different due to discretization effects. Each time one section of the pipe goes from liquid to two-phase it generates a small pressure shock wave in the system. When there are pressure oscillations the phase of the sections is constantly changing and thus high-frequency shock waves are generated, superimposed on the slower pressure oscillations, see Figure 7. These waves could be mistaken for high-frequency density-wave oscillations, (8), but are really discretization arte-facts. One way around this problem would be to use a so-called moving boundary model, (3), but this has not been done here.
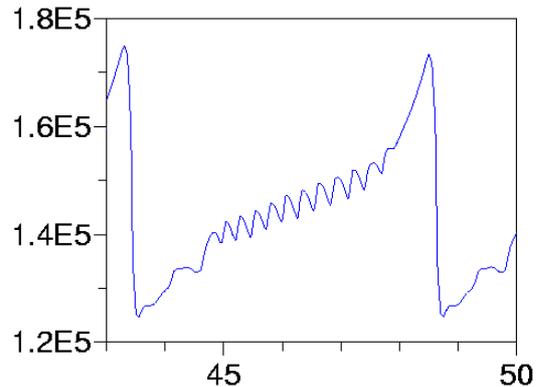
Fig. 7 High-frequency oscillations with fully discretized model.

5. Comparisons

The pressure drop oscillations observed using the simple model from Section 2 are similar in period and amplitude to the oscillations obtained with the one-flow discretized model. Note that the shape of the pressure drop curve, and thus the properties of the oscillations, depend very much on the average media properties used, ρ_l, ρ_v, h_l, h_v and h_{in}. In the complex model the properties change with the pressure and mass flow into the system, which causes differences in limit cycle period and amplitude. In Figure 8 the pressure drop characteristic is plotted together with limit cycles obtained with the one-flow model. The amplitude and damping of the oscillations vary with the mean flow. This is caused by the energy dynamics which produce a lag in the density changes, unlike the immediate response of the pressure drop function in the simplified model. The simple model uses constant, average properties and thus gives the same limit cycle amplitude for all mass flows within the unstable region.

6. Conclusions

A simplified model of a boiler tube has been derived, assuming equilibrium conditions and a linear quality profile also during transients. The model gives a closed expression for the pressure
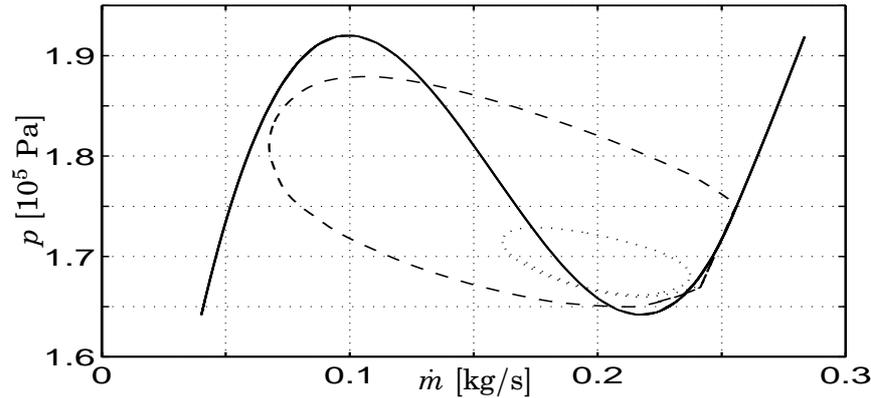
Fig. 8 Simulation results of the complex model, characteristic obtained from the simplified model. Two limit cycles for different mean flows, $\dot{m} = 0.15$ kg/s ($--$) and 0.2 kg/s (- - -).

drop which depends on mass flow through the tube and the ratio of vapor and liquid density. The simplified model gives insight into how a known instability phenomenon, pressure-drop oscillations, arises. The simplified model also gives results close to measurements in (5).

The simplified model has also been compared to two different discretized models developed in Modelica. A one-flow model with lumped mass balance and discretized energy balance and a fully discretized model. The one-flow model is shown to give more realistic oscillations than the simplified model. The fully discretized model, however, gives high frequency oscillations due to discretization effects and is not reliable for studies of pressure-drop oscillations.

The two discretized models was built using a thermo-hydraulic base library in Modelica, ThermoFlow. The example shows how models for studying a complicated phenomenon can be built from model library units, and how the library units can be adapted.

References

[1] C. J. Aldridge and A. C. Fowler. "Stability and instability in evaporating two-phase flows." *Surveys on Mathematics for Industry*, **6**, pp. 75–107, 1996.

[2] R. D. Bell and K. J. Åström. "Drum-boiler dynamics." *Automatica*, **36:3**, pp. 363–378, 2000.

[3] P. A. Heusser. *Modelling and Simulation of Boiling Channels with a General Front Tracking Approach*. SCS, 1996.

[4] S. Kakaç and H. T. Liu. "Two-phase flow dynamic instabilities in boiling systems." In Chen *et al.*, Eds., *Proc. 2nd Int. Symp. on Multi-Phase Flow and Heat Transfer*, vol. 1, pp. 403–444, 1991.

[5] H. T. Liu, H. Koçak, and S. Kakaç. "Dynamical analysis of pressure-drop type oscillations with a planar model." *Int.J. Multiphase Flow*, **21:5**, pp. 851–859, 1995.

[6] H. Tummescheit. "Object-oriented modeling of physical systems, part 11." *Automatisierungstechnik*, **48:2**, 2000. In german.

[7] H. Tummescheit, J. Eborn, and F. Wagner. "Development of a Modelica base library for modeling of thermo-hydraulic systems." In *Modelica 2000 Workshop Proceedings*, Lund, October 2000. Modelica Association.

[8] G. Yadigaroglu. "Two-phase flow instabilities and propagation phenomena." In Delhaye *et al.*, Eds., *Thermal-hydraulics of Two-phase Systems for Industrial Design and Nuclear Engineering*, chapter 17. Hemisphere McGraw-Hill, New York, 1981.