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Abstract

The development process for spacecraft control systems relies heavily on modelling and simulation tools for spacecraft dynamics. For this reason, there is an increasing need for adequate design tools in order to cope efficiently with tightening budgets for space missions. In this paper, the main issues related to the modelling and simulation of satellite dynamics are briefly summarised, and the results obtained so far in developing Modelica tools for spacecraft simulation are presented and illustrated with a case study for a satellite equipped with Control Moment Gyros as main attitude control actuators.

1 Introduction

The safe and satisfactory operation of a satellite, in terms of its mission objectives, is strongly related to the performance level of its on-board attitude and orbit control systems, which provide the ability to maintain a desired orientation in space (or, e.g., carry out predefined attitude maneuvers) and track a desired, nominal orbit in spite of the presence of external disturbances. In addition, the recent trend towards missions based on constellations or formations of small satellites has led to the formulation of even more complex control problems, related to the relative motion (both in terms of attitude and position) of more vehicles at a time. However, spacecraft designers are also faced with a general reduction of space programmes budget, especially for scientific Low Earth Orbit (LEO) missions, embodied by the spreading of the "faster, better, cheaper" philosophy. This has resulted in an increasing need for efficient design tools in every domain involved in spacecraft design, and particularly in the area of control oriented modelling and simulation. Specific tools have to be developed for the design of both the system architecture and the Attitude and Orbit Control System (AOCS), bearing in mind the principles of reusability, flexibility and modularity. The main issue in the development of such tools should be to try and work out a unified environment to be used throughout the life cycle of the AOCS software, namely, the mission analysis stage, the preliminary and detailed design and simulation phases, the generation and testing of the on-board code, the development of the AOCS Electrical Ground Support Equipment (EGSE) and the post-launch data analysis activities. A number of commercial tools are available to support one or more of the above mentioned phases in the development of AOCS subsystems, however none of them seems capable of providing complete coverage of the whole development cycle in a sufficiently flexible way.

In particular, the experience gathered in the development of control-oriented spacecraft modelling tools within a "signal oriented" simulation environment (see [2]) showed that a more systematic approach, based on modern acausal object-oriented modelling languages such as Modelica (see [3, 6]), might lead to the development of a spacecraft simulation library the use of which would be made much more efficient by the very nature of the selected modelling approach. Note, in passing, that there is an increasing interest for multidomain problems in the spacecraft control design community (see, e.g., [17]), an area which would benefit from the availability of simulation tools based on the object-oriented approach.

Surprisingly enough, while the use of Modelica for aerospace applications has recently led to the development of a library for flight dynamics (see [14]), very little activity in the spacecraft domain has been reported yet. The development of simulation tools for satellite attitude and orbit dynamics within the object-oriented paradigm has been the subject of previous work (see [10]). Since the development of the model components presented in the cited references, however, a new, more refined version of the Modelica

Multibody library has been released (see [15]) which turns out to be extremely suitable to serve as a basis for the development of the basic model components for the mechanical parts of spacecraft models. In particular, the adoption of the above mentioned library would prove specially beneficial for the simulation of spacecraft equipped with momentum exchange devices, such as, e.g., control moment gyros (CMGs, see, e.g., [8, 22]).

Therefore, the aim of the paper will be to present the current state of the development of spacecraft modelling tools based on the Multibody library, with specific reference to the problem of analysing the (open and closed loop) attitude of satellites equipped with control moment gyros (CMGs) as main attitude control actuators.

The paper is organised as follows: first a brief introduction to the role of mathematical modelling and simulation in the development cycle of spacecraft control system is given, in Section 2; subsequently, an overview of the main model components involved in typical control oriented models will be presented. Finally, the proposed approach to spacecraft modelling will be described in Sections 3-7 and the results obtained in the implementation and application of such an approach to the simulation of spacecraft equipped with CMGs will be presented and discussed in Section 8.

2 Modelling and simulation for AOCS design

As mentioned in the previous Section, the development of the AOCS subsystem for a satellite can be decomposed in the following phases:

1. Feasibility study (conceptual design).
2. Preliminary design.
3. Detailed design.
4. Code generation and testing.

During each of these phases, the designer should be able to rely on appropriate modelling and simulation tools. In particular, modelling tools should be flexible enough to provide the required level of complexity during each of the development phases.

For example, consider the tasks for which a simulation tool would be applied during the feasibility study phase (see Figure 1 and the classical reference [9]):

- Attitude control strategy definition, starting from mission requirements and platform characteristics;
- Evaluation of the external force and torque disturbances acting on the spacecraft, depending on the mission profile. This is normally done using a simple attitude control algorithm to maintain the satellite at nominal conditions;
- Selection and sizing of the actuators in order to counteract disturbances and to maintain the nominal pointing accuracy required by the mission;
- Verification of the possibility to fulfill possible maneuver requirements with the selected actuators.

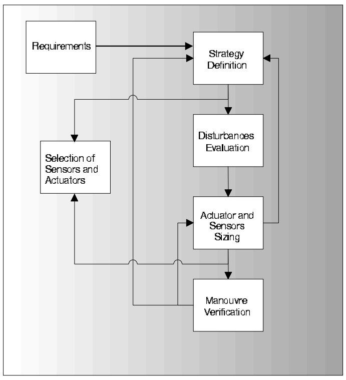

Figure 1: Block diagram of preliminary ACS design process.

The above tasks can be performed without resorting to a dynamic simulator for the spacecraft, however they require accurate modelling capability for orbit dynamics and the availability of reliable models for the space environment. As an example, consider the problem of assessing the external torques acting on the spacecraft. Clearly their characteristics will depend also on the selected control strategy (actuators, sensors and control laws) for the spacecraft, which however is not entirely defined at this stage. Therefore only nominal and/or worst case scenarios need be considered. On the other hand, all the subsequent design phases require the possibility of integrating orbit models and environmental models with a fully dynamic spacecraft simulator, in order to proceed to the refinement of the original design concept. In particular, as the development process goes on, more and more accuracy in the prediction of the achievable performance is required, so that the complexity of the simulation environment is progressively increased.

3 The Modelica Spacecraft modelling library

A library of tools for the modelling and simulation of spacecraft dynamics based on the Modelica language is currently being developed. Modelica turns out to be specially suited for the modelling of spacecraft dynamics under many respects:

- Coordinate frames can be simply included in the model in terms of connectors, describing kinematic transformations from one coordinate system to another.
- Spacecraft dynamics is modelled by defining a Spacecraft class which can be (almost) directly implemented in terms of classical equations for rigid body motion. The data structure to be used in representing all the quantities involved in a specific spacecraft model arises naturally during the modelling process.
- Specific Modelica constructs are available to deal with the modelling of physical fields and environmental quantities. This feature turns out to be extremely useful in modelling the space environment and representing the interaction between the environment and the spacecraft. In particular, with a suitable choice of the environment interfaces, models of increasing complexity for each of the quantities described in Section 5 can be defined. This feature allows for a simple and very convenient implementation of the ”scalability” requirement formulated in Section 2.
- Sensors and actuators can also be easily represented in the Modelica paradigm. For example, the generation of magnetic torques is modelled in terms of the interaction with the geomagnetic field, while the momentum exchange between spacecraft and wheels is modelled via a simple mechanical connector allowing one rotational degree of freedom [1].
- Packages of data sheets for each class can be constructed and components easily modified within each spacecraft model, using Modelica’s advanced features (see, e.g., [16]).
- Finally, as the components of the library are independent from each other, one can exploit this flexibility in order to build a simulation model of increasing complexity and accuracy according to the needs associated with each phase of the AOCS development process. As an example, one can carry out an analysis of the external disturbance forces and torques acting on the spacecraft in its nominal orbit and attitude, by defining a ”simplified” spacecraft ideally attached to its nominal reference attitude.

The original approach to the development of the library contemplated the development of dedicated components also for the mechanical parts. However, the availability of the recently upgraded (see [15]) Multibody Library is leading to some significant changes, since the reuse of the Multibody components would lead to some significant advantages.
The main components of the library are the following:

- A set of basic functions for operations on orbit parameters (transformations between cartesian and orbit elements, see for example [13, 19].
- A similar set of functions for operation on attitude parameters (attitude matrix, quaternions, Euler angles). These have been partially based on the Rigid Body Kinematics Toolbox (see [18]).
- Class definitions for Planet, Orbit, Spacecraft, and the most commonly used actuators and sensors.
- Environmental models of various complexity for gravitational and magnetic field.
- Data sheets for basic model components, such as orbits, actuators and sensors.

4 Dynamics of a spacecraft equipped with momentum exchange devices

For the purpose of the present analysis, the following reference systems are adopted:

- Earth Centered Inertial reference axes (ECI). The origin of these axes is in the Earth’s centre. The X-axis is parallel to the line of nodes, that is the intersection between the Earth’s equatorial plane and the plane of the ecliptic, and is positive in the Vernal equinox direction (Aries point). The Z-axis is defined as being parallel to the Earth’s geographic north-south axis and pointing north. The Y-axis completes the right-handed orthogonal triad.

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[1] Mounting errors, which may give rise to interaxis coupling and vibrations, can be easily accounted for.

- Earth Centered Fixed reference axes (ECF).
- Pitch-Roll-Yaw axes. The origin of these axes is in the satellite centre of mass. The X-axis is defined as being parallel to the vector joining the actual satellite centre of gravity to the Earth's centre and positive in the same direction. The Y-axis points in the direction of the orbital velocity vector. The Z-axis is normal to the satellite orbit plane and completes the right-handed orthogonal triad.
- Satellite body axes. The origin of these axes is in the satellite centre of mass; the axes are assumed to coincide with the body's principal inertia axes.

The equations of rotational motion of a rigid spacecraft equipped with momentum-exchange actuators such as CMGs are given by

$$\dot{H} + \omega \times H = T_{ext} \tag{1}$$

where $H = (H_1, H_2, H_3)^T$ is the angular momentum vector of the whole system expressed in the spacecraft body-fixed control axes; $\omega = (\omega_1, \omega_2, \omega_3)^T$ is the spacecraft angular velocity vector; T_{ext} is the global external torque vector applied to the spacecraft, including gravity gradient, solar pressure and aerodynamic torques, expressed in the body-fixed control axes.

The total angular momentum vector consists of the spacecraft main body angular momentum and the angular momentum of the exchange devices, that is

$$H = J\omega + h \tag{2}$$

where J is the overall inertia matrix of the spacecraft and h is the total CMG momentum vector expressed in the body-fixed control axes.

Combining Eqs. (1) and (2), we obtain

$$(J\dot{\omega} + \dot{h}) + \omega \times (J\omega + h) = T_{ext} \tag{3}$$

or, introducing the internal control torque vector generated by CMGs $\tau = -(\dot{h} + \omega \times h)$, we can rewrite Eq. (3) as

$$J\dot{\omega} + \omega \times J\omega = \tau + T_{ext} \tag{4}$$

In addition to the dynamic equations of motion, kinematic equations must be included in the model, which can be easily parameterised in terms of the attitude quaternion

$$\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \\ \dot{q}_4 \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 0 & \omega_3 & -\omega_2 & \omega_1 \\ -\omega_3 & 0 & \omega_1 & \omega_2 \\ \omega_2 & -\omega_1 & 0 & \omega_3 \\ -\omega_1 & -\omega_2 & -\omega_3 & 0 \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix} \tag{5}$$

5 Single Gimbal Control Moment Gyros

A gyroscope, or gyro, is any instrument which uses a rapidly spinning mass to sense and to respond to changes in the inertial orientation of its spin axis. Three basic types of gyroscopes are used on spacecraft: *rate gyros* (RGs) and *rate integrating gyros* (RIGs) are attitude sensors used to measures changes in the spacecraft orientation; *control moment gyros* (CMGs) are used to generate control torques to change and maintain the spacecraft orientation.

A single gimbal control moment gyro (SGCMG) consists of a rotor spinning at a constant rate around an axis that is gimballed to allow changes in its spin direction. The gimbal is rigidly attached to the spacecraft, so that torques generated in response to its input axis rotation apply to the spacecraft itself.

Let $\hat{\theta}_i$ the unit vector along the i-th SGCMG gimbal axis, $\hat{h}_i$ the unit vector along the instantaneous angular momentum, $\hat{j}_i = \hat{\theta}_i \times \hat{h}_i$

Each angular momentum vector depends upon the relevant gimbal angle (for its direction). With respect to the reference frame (spacecraft body axes), the total angular momentum for a system of n SGCMGs is the vector sum of the individual momenta:

$$h(\theta) = \sum_{i=1}^{n} h_i(\theta_i) = f(h_i, \theta_i, \beta_i) \tag{6}$$

A typical arrangement for SGCMGs is the one in which the CMGs are constrained to gimbal on the faces of a pyramid and the gimbal axes are orthogonal to the pyramid faces. In this case, the overall angular momentum is given by

$$\begin{aligned} h(\theta) = h_1 \begin{bmatrix} -\cos\beta\sin\theta_1 \\ \cos\theta_1 \\ \sin\beta\sin\theta_1 \end{bmatrix} + h_2 \begin{bmatrix} -\cos\theta_2 \\ -\cos\beta\sin\theta_2 \\ \sin\beta\sin\theta_2 \end{bmatrix} + \\ + h_3 \begin{bmatrix} \cos\beta\sin\theta_3 \\ -\cos\theta_3 \\ \sin\beta\sin\theta_3 \end{bmatrix} + h_4 \begin{bmatrix} \cos\theta_4 \\ \cos\beta\sin\theta_4 \\ \sin\beta\sin\theta_4 \end{bmatrix} \end{aligned} \tag{7}$$

where β is the skew angle of the pyramid and $h_1 = h_2 = h_3 = h_4$ for the considered cluster of four pyramid-mounted SGCMGs. In particular, when each CMG has the same angular momentum about its spin-rotor axis and the skew angle is chosen as $\beta = 54.73\,\text{deg}$, the momentum envelope becomes nearly spherical.

The total amount of angular momentum for the system is limited both in value and direction, by individual

SGCMGs momenta. The time derivative of the global CMG angular momentum vector con be obtained as

$$\dot{h}(\theta,\dot{\theta}) = \sum_{i=1}^{4} \dot{h}_i(\theta_i,\dot{\theta}_i) = [A(\theta_i)]\dot{\theta} \tag{8}$$

where $\theta = (\theta_1,\theta_2,\theta_3,\theta_4)^T$ is the gimbal angle vector and A is the instantaneous 3×4 Jacobian Matrix.
The gimbal rate command $\dot{\theta}$ is derived in such a way as to supply the required angular momentum for control purposes. A frequently adopted approach is to note that $\dot{\theta}$ can be obtained as

$$\dot{\theta} = A^{\dagger}\dot{h} \tag{9}$$

where by $A^{\dagger} = A^T(AA^T)^{-1}$ we denote the Moore-Penrose pseudoinverse of the Jacobian matrix A.

6 Control system

6.1 Attitude control

Since the torque-producing gimbal rates provided by equation (9) can lead to significant problems in the operation of the control law whenever the configuration of the CMGs is such that the Jacobian matrix A is nearly singular, a new strategy must be adopted invoking an approximate solution to equation (8), which must be capable of both minimizing the errors introduced in the output torque supplied and steering the system away from singular states configuration neighborhoods. The errors introduced by this approach in the torque supplied by the SGCMGs cluster can be dealt by the control system as disturbances torques, and appropriately compensated.
In order to determine an inverse solution to equation (8) even when the rank of A is less than 3, the Singularity Robust Inverse solution obtained by solving the following minimization problem must be invoked:

$$\min \frac{1}{2} e^T W e \tag{10}$$

$$e = \begin{bmatrix} \dot{h} - A\dot{\theta} \\ \dot{\theta} \end{bmatrix}$$

$$w = \begin{bmatrix} P & 0 \\ 0 & Q \end{bmatrix}$$

where P and Q are positive definite weighting matrices, that is, $P = P^T > 0$ and $Q = Q^T > 0$.
Minimizing for $\dot{\theta}$, the singularity robust inverse solution can be obtained as

$$\dot{\theta} = A^{\#}\dot{h} \tag{11}$$

where $A^{\#} = [A^T P A + Q]^{-1} A^T P$. Note that $[A^T P A + Q] > 0$ and, thus, nonsingular for any set of gimbal angles.
If $Q = 0$, the singularity robust inverse solution has the form of the classical, weighted least squares solution which exists only for a full rank Jacobian matrix A.

6.2 Momentum management

The gimbal angles of a CMG equipped spacecraft may drift to various non-optimal values, due to external disturbances. This can force the spacecraft into a singular state or into a lack of control authority. Typically, a periodic disturbance torque along one spacecraft axis would result in a cyclic variation in the angular velocity of the actuation device directed along that axis, while a constant (secular) disturbance would lead to a linear increase in angular velocity, as the relevant CMG gimbal angle would be accelerated at a constant rate in order to transfer to it the excess of angular momentum due to the external disturbance. This can be sustained up to the physical limit for the rotational speed of the device. In order to prevent this limit from being reached, the so-called desaturation of the actuator must be performed, i.e. an extra set of actuators, generating external torques, must be used to dump momentum from the spacecraft.
Basically, the idea is to use the CMGs cluster to control the spacecraft as required by the Attitude Control System, and to achieve their desaturation by means of a dedicated controller, that keeps the CMGs gimbal rates as small as possible. The goal of the momentum control loop is to maintain the CMGs momentum near zero without interfering with the attitude control loop, that is, the momentum control loop must have a considerably slower response with respect to the attitude control loop.
The external torque to be applied to the spacecraft required to compensate for the gimbal angle offset is taken as

$$T_r = -k\,h_{cmgs} = -A(\theta)\dot{\theta}_e \tag{12}$$

where

$$\dot{\theta}_e = \frac{\theta - \theta^*}{\Delta t}$$

θ^* is the gimbal angle reference vector and Δt is the time it takes the current gimbal angles to converge to the reference gimbal angles (response time of the momentum control loop).
External torques may be generated by such devices as thrusters or magnetic coils. Since the compensation of the external disturbances is better handled continuously, and given the usual restrictions on waste of

consumable in space applications, magnetic control is usually preferred.

A set of three magnetic coils, aligned with the spacecraft principal inertia axes generate torques according to

$$T_{coils} = m_{coils} \times b \tag{13}$$

where $m_{coils} \in \mathbb{R}^3$ is the vector of magnetic dipoles for the three coils, representing the actual control variables for the coils, and $b(t) \in \mathbb{R}^3$ is the Earth magnetic field described in the body reference frame. The dynamics of the electric coils reduce to a very short electrical transient, and as such can be neglected.

By equating Eqs. (12), (13) and left multiplying by $b(t)$, the magnetic dipole to be applied by the coils and the resulting torque may be derived

$$m_{coils} = -\frac{b \times (A(\theta)\,\dot{\theta}_e)}{||b||^2} \tag{14}$$

$$T_{coils} = -\frac{b \times (A(\theta)\,\dot{\theta}_e)}{||b||^2} \times b \tag{15}$$

7 Model components for CMGs

Taking advantage of the recently released New Modelica Multibody library (see [15]), a set of simulation tools has been developed for satellite attitude and orbit dynamics. Specifically, the following components have been developed:

- Extended World Model: a new World model, extending Modelica.MultiBody.World has been defined. It accounts for a more refined description of the Earth's gravitational potential and introduces a model for the geomagnetic field. Such an extension to the basic World model provided in the Multibody library plays a major role in the realistic simulation of the dynamics of a spacaecraft as the linear and angular motion of a satellite are significantly influenced by its interaction with the space environment.

- Extended rigid body model: similarly, a new rigid body model, extending Modelica.MultiBody.Parts.Body has been defined. The main modification is the possibility of taking into account the interaction between the spacecraft and the geomagnetic field, i.e., to model the effect of magnetic torques applied on the satellite while orbiting around Earth.

- Rotor: Model of a Single Gimbal Control Moment Gyro, including as input the gimbal angular rate (feeded by the attitude control system) and as output the gimbal angle. Developed using standard Modelica Multibody Library components.

- Cluster: Model including the classical set of four pyramid-mounted Single Gimbal Control Moment Gyros arrangement. Developed using standard Modelica Multibody Library components.

- Attitude Control System: Block computing the required gimbal rates for the four Single Gimbal Control Moment Gyros used as control torque actuating devices.

For each model component, a short description is given in the following subsections.

7.1 Extended World model

The Extended World Model is an extension of the former Modelica.MultiBody.World model, including among the available selections a more sophisticated model for the Earth's gravity field (described up to the J_2 term of the Earth's gravitational potential) and a model for Earth's magnetic field (modelled up to the quadrupole terms).

As a consequence of Earth's oblateness and not homogeneity, the geomagnetic and gravitational potentials are a non linear function of both the point latitude and longitude, in addition to the distance from the center of the Earth.

The Earth's gravitational potential U_g may be described by the function

$$U_g(r,\theta,\lambda) = -\frac{\mu}{r}\{1 + \sum_{n=2}^{\infty}(\frac{R_e}{r})^n J_n P_n(cos(\theta)) +$$
$$+\sum_{n=2}^{\infty}\sum_{m=1}^{n}(\frac{R_e}{r})^n P_n^m(cos(\theta))(C_n^m cos(m\lambda) + S_n^m sin(m\lambda))\}$$

where P_n^m are the Legendre polynomials

$$P_n(x) = \frac{1}{2^n n!}\frac{d^n}{dx^n}(x^2-1)^n$$

$$P_n^m(x) = (1-x^2)^{m/2}\frac{d^m P_n(x)}{dx^m}$$

R_e is the mean equatorial Earth radius, r,θ and λ are the point's spherical coordinates and coefficients J_n, C_n^m, S_n^m are the zonal, sectoral and tesseral coefficients. For the purpose of attitude control simulations a satisfactory approximation can be obtained by neglecting

n	m	g [nT]	h [nT]
1	0	-29682	
1	1	-1789	5318
2	0	-2197	
2	1	3047	-2356
2	2	1685	-425

Table 1: Coefficients of the geomagnetic field model.

the terms after J_2. The Earth gravitational field components (expressed in spherical coordinates) are then given by

$$g = -\nabla U_g = -\{\frac{\partial U_g}{\partial r}, \frac{1}{r}\frac{\partial U_g}{\partial \theta}, \frac{1}{r\,sin(\theta)}\frac{\partial U_g}{\partial \lambda}\}. \quad (16)$$

As for the geomagnetic potential U_m, it is described by the function

$$U_m(r,\theta,\lambda) = \frac{R_e}{\mu}\sum_{n=0}^{\infty}\sum_{m=0}^{n}(\frac{R_e}{r})^{n+1}P_{nm}(cos(\theta))$$
$$(g_n^m cos(m\lambda) + h_n^m sin(m\lambda))$$

where g_n^m and h_n^m are the Gauss coefficients appropriate to the Schmidt polynomials P_{nm}

$$P_{n,0}(x) = P_n^0(x)$$
$$P_{n,m}(x) = (\frac{2(n-m)!}{(n+m)!})^{1/2}P_n^m(x).$$

A first approximation for the geomagnetic potential is obtained by neglecting the terms after the quadrupole. The coefficients for the geomagnetic potential adopted in the simulation environment correspond to the so-called International Geomagnetic Reference Field (IGRF) model for the Earth's magnetic field and are given in Table 1. The components of the geomagnetic field (expressed in spherical coordinates) are then given by

$$B = -\nabla U_m = -\{\frac{\partial U_m}{\partial r}, \frac{1}{r}\frac{\partial U_m}{\partial \theta}, \frac{1}{r\,sin(\theta)}\frac{\partial U_m}{\partial \lambda}\}. \quad (17)$$

The new function magneticField, embedded in the World model, receives as input the body position and provides as outputs the corresponding components of the geomagnetic field vector B, expressed in the ECI reference frame. Since the geomagnetic field model is defined with respect to the rotating ECF reference frame, an additional parameter UT_0, defining the initial value for the Universal Time (UT) and whose default value is set to zero, allows to start the simulation with a desired initial rotation of the ECF reference frame with respect to the ECI reference frame.

7.2 Extended rigid body model

This new component extends the Modelica.MultiBody.Parts.Body model to account for the effects of the Earth's magnetic field in terms of torques applied to the satellite. Indeed, if the spacecraft possesses a magnetic dipole m, it experiences an external torque T given by

$$T = m \times B, \quad (18)$$

where B is the geomagnetic field vector in body coordinates. Note, in passing, that the ability of taking into account in the rigid body model the interaction with the geomagnetic field makes it possible to fulfill two different modelling goals: first of all to simulate the effect on the satellite dynamics of a *residual magnetic dipole*, such as due to the internal magnetic field generated by on-board electrical equipment; furthermore, it is possible to model the effect on the angular motion of the spacecraft of *magnetic actuators*, which, as previously mentioned, are frequently used, specially in small satellite missions, for attitude or momentum management.

7.3 Rotor

The SpacecraftDynamicsLibrary.Rotor model simulates a Single Gimbal Control Moment Gyro, which has been chosen as the primary actuation source for the satellite attitude manoeuvering and control. The following standard Modelica library components have been employed:

- MultiBody.Interfaces.Frame_a, used as the SGCMG-satellite connecting point.
- MultiBody.Joints.ActuatedRevolute, used to model the SGCMG rotational degree of freedom. The gimbal speed is driven through a Mechanics.Rotational.Speed by the input signal coming from the satellite attitude control system (SpacecraftDynamicsLibrary.ACS block).
- Two Mechanics.Rotational.Speed
- Blocks.Interfaces.InPort, feeding to the SGCMG the desired gimbal rate.
- Mechanics.Rotational.Sensors.SpeedSensor
- MultiBody.Parts.FixedTranslation
- MultiBody.Parts.Rotor1D, used as the physical rotor. Its speed is kept constant by means of

an electrical motor, whose dynamic has not been modelled at this stage, and which is represented by a Blocks.Sources.Constant. The rotor rotational inertia has been assigned such that the resulting angular momentum along the rotor axis is $1.76Nms$.

- Blocks.Continuous.Integrator
- Blocks.Sources.Constant, forcing the rotor angular rate to the chosen 4000 rpm nominal value.
- Blocks.Interfaces.OutPort, used to output to the satellite attitude control system (SpacecraftDynamicsLibrary.ACS block) the actual gimbal angle.

7.4 Attitude Control System

The SpacecraftDynamicsLibrary.ACS block computes the four SGCMGs gimbal rates required to control the satellite attitude.

The computation is performed in two separate steps. First, the required control torques are computed, by means of a suitable state feedback gain (designed via LQR control techniques), where the system states considered in the control algorithm are the quaternion errors and the satellite angular rates errors. Subsequently, the gimbal rates are derived in such a way as to supply the required torques (via variation of the SGCMGs cluster angular momentum). To this purpose, the control moment gyro steering logic proposed by Wie, Bailey and Heiberg [22] was adopted.

The SpacecraftDynamicsLibrary.ACS provides the four SGCMGs gimbal rates, and receives as inputs the gimbal angles, the satellite inertial attitude quaternion, the measured gimbal rates and the unit vectors of the local orbital frame.

7.5 Cluster

The SpacecraftDynamicsLibrary.Cluster model simulates the classical configuration of four identical pyramid mounted Single Gimbal Control Moment Gyros, with a skew angle $\beta = 54.73^o$. It employs four SpacecraftDynamicsLibrary.Rotor models.

Receives as input the SGCMG gimbal rates computed by the SpacecraftDynamicsLibrary.ACS block.

8 Simulation study

In order to analyze the performance of the spacecraft dynamics library components developed, a specific mission scenario has been considered, namely:

- Spacecraft in equatorial orbit (0^o inclination).
- Orbit altitude of 450 Km (a typical altitude for a small scientific mission).
- The attitude control must keep an Earth pointing attitude, aligned with the Pitch-Roll-Yaw reference frame (orbital frame).
- The spacecraft is provided with a star sensor for attitude determination (i.e., an ideal state feedback situation is considered).
- Attitude control is based on a set of four pyramid mounted Single Gimbal Control Moment Gyros.
- The spacecraft is subject to the effect of a magnetic disturbance torque, due to a residual magnetic dipole for which a value of $1Am^2$ along each body axes has been assumed.

As an illustrative example, a simulation has been carried out in which the initial conditions for the spacecraft are characterised by a high value of the components of the angular rate, such as would occur during the initial acquisition of the desired Earth pointing attitude. Figure 2 shows how three axis attitude control is achieved by means of the four SGCMG actuators. In particular, note that the residual pointing error due to the geomagnetic disturbance torque is hardly visible. The corresponding time history of the geomagnetic field along the considered orbit and of the associated disturbance torque are shown in Figure 3. As expected, since the considered spacecraft is operating in an equatorial orbit, the only significant component of the geomagnetic field is aligned with the orbit normal (Z ECI axis). In spite of this, it is interesting to note that because of equation (18) the spacecraft is subject to a disturbance torque along all three axes.

As was mentioned previously, external disturbance torques force the gimbal angles of the CMGs to slowly drift away from their optimal value. In particular, the periodic component of the magnetic torques force a cyclic variation in the angular rate of the SGCMGs, while the secular component lead to a linear increase in the gimbal angular rates. This can be sustained up to the physical limit for the rotational speed of the device. In order to prevent this limit from being reached, the actuator must be desaturated, i.e. an extra set of actuators, generating external torques, must be used to dump momentum from the spacecraft.

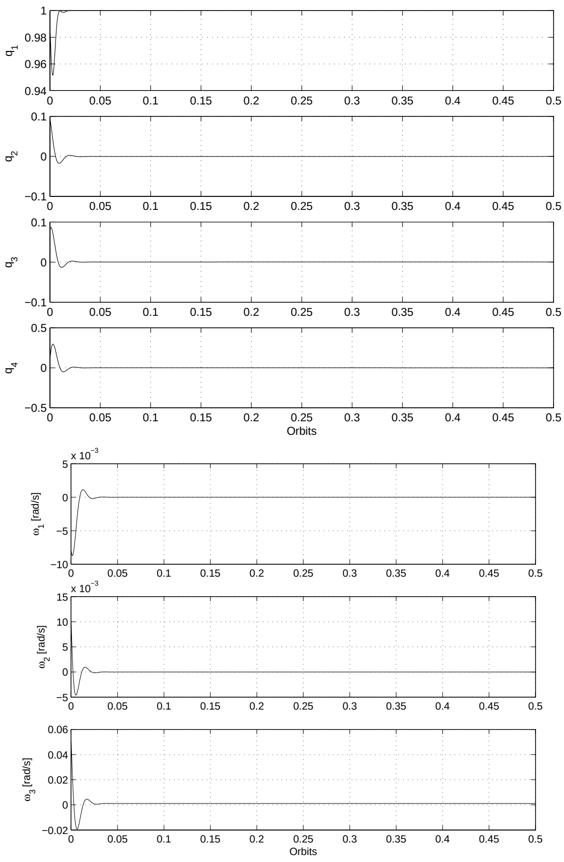

Figure 2: Quaternion and angular rates for the attitude acquisition.

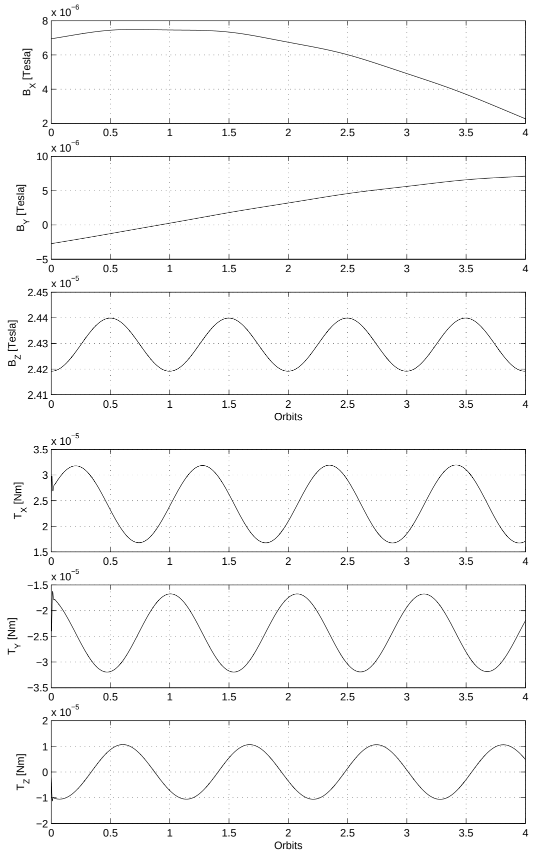

Figure 3: Geomagnetic field and magnetic disturbance torque along the considered orbit.

9 Concluding remarks

The main issues related to the modelling and simulation of satellite dynamics have been described, the results obtained so far in developing Modelica tools for spacecraft simulation have been presented and a case study for a satellite equipped with Control Moment Gyros as main attitude control actuators has been illustrated by means of a simulation study.

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References

[1] Alberti, M., De Rocco, L., Morea, D., and Lovera, M. Environmental external torques estimation of a low Earth orbiting satellite based on MITA platform flight data. In: 52nd IAF International Astronautical Congress, Toulouse, France, 2001.

[2] Annoni, G., De Marchi, E., Diani, F., Lovera, M. and Morea, G.D. Standardising tools for attitude control system design: the MITA platform experience. In: Data Systems in Aerospace (DASIA) 1999, Lisbon, Portugal, 1999.

[3] Modelica Association, Modelica - a unified object-oriented language for physical systems modelling. Language specification. Technical report, `http://www.modelica.org`, 2002.

[4] Bate, R.R., Mueller, D.D. and White, J.E., Fundamentals of astrodynamics. Dover Publications Inc., 1971.

[5] Della Torre, A., Lupi, T., Sabatini, P. and Coppola, P. MITA: the Advanced Technology Italian Minisatellite.In: Data Systems in Aerospace (DASIA) 1999, Lisbon, Portugal, 1999.

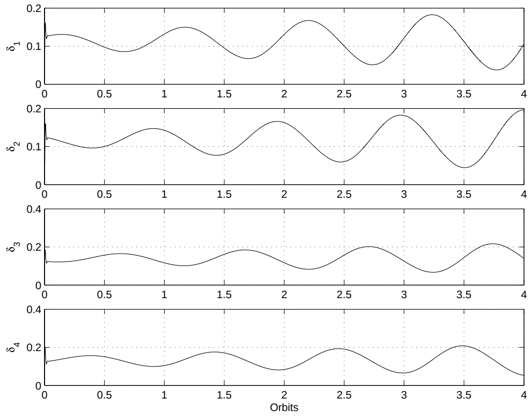

Figure 4: Gimbal angles for the attitude acquisition.

[6] Fritzson, P. and Bunus, P. Modelica - a general object-oriented language for continuous and discrete-event system modelling and simulation. In: Proceedings of the 35th IEEE Annual Simulation Symposium, San Diego, CA, 2002.

[7] Hughes, P. Spacecraft attitude dynamics. John Wiley and Sons, 1986.

[8] Lappas, V. J., Steyn, W. H., and Underwood, C.I. Practical Results on the Development of a Control Moment Gyro based Attitude Control System for Agile Small Satellites. In: Proceedings of the 16th AIAA/USU Small Satellite Conference, Session SSC02-VIII-8, 2002.

[9] Larsen, W.J. and Wertz, J.R., editors. Space mission analysis and design. Kluwer Academic Publisher, 1992.

[10] Lovera, M. Object-oriented modelling of spacecraft attitude and orbit dynamics. In: Proceedings of the 54th International Astronautical Congress, Bremen, Germany, 2003.

[11] Lovera, M. and Astolfi, A. Spacecraft attitude control using magnetic actuators, Automatica, 40(8):1405–1414, 2004.

[12] Martinez-Sanchez, M. and Pollard, J.E. Spacecraft electric propulsion - an overview. Journal of Propulsion and Power, 14(5):688–699, 1998.

[13] Montenbruck, O. and Gill, E. Satellite orbits: models, methods, applications. Springer, 2000.

[14] Moorman, D. and Looye, G. The Modelica flight dynamics library. In: Proceedings of the 2nd International Modelica Conference, Oberpfaffenhofen, Germany, 2002.

[15] Otter, M., Elmqvist, H. and Mattsson, S. E. The New Modelica MultiBody Library. In: Proceedings of the 3nd International Modelica Conference, Linköping, Sweden, 2003.

[16] Otter, M. and Olsson, H. New features in Modelica 2.0. In: Proceedings of the 2nd International Modelica Conference, Oberpfaffenhofen, Germany, 2002.

[17] Roithmayr, C., Karlgaard, C., Kumar, R. and Bose, D. Integrated Power and Attitude Control with Spacecraft Flywheels and Control Moment Gyroscopes. Journal of Guidance, Control, and Dynamics, 27(5):859-873, 2004.

[18] Schaub, H. and Junkins, J.L. Matlab toolbox for rigid body kinematics. In: AAS/AIAA Space Flight Mechanics Meeting, Breckenridge, CO, 1999.

[19] Sidi, M. Spacecraft dynamics and control. Cambridge University Press, 1997.

[20] Silani, E. and Lovera, M. Magnetic spacecraft attitude control: A survey and some new results. Control Engineering Practice, 13(3):357-371, 2005.

[21] Wertz. Spacecraft attitude determination and control. D. Reidel Publishing Company, 1978.

[22] Wie, B., Bailey, D., and Heiberg, C. Singularity Robust Steering Logic for Redundant Single-Gimbal Control Moment Gyros. Journal of Guidance, Control, and Dynamics, 24(5):865-872, 2001.